If we apply this to second derivatives, we find $D_x^2 w = D_u^2 w (D_x^1 u)^2 + D_u^1 w D_x^2 u$.